Inclusion of non-spherical components of the Pauli blocking operator in (p,p′) reactions

E. J. Stephenson
Indiana University Cyclotron Facility
Bloomington, IN 47408

R. C. Johnson
Department of Physics, University of Surrey
Guildford, Surrey GU2 5XH, UK

F. Sammarruca
Department of Physics, University of Idaho
Moscow, Idaho 83844

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Abstract

We present the first calculations of proton elastic and inelastic scattering in which the Pauli blocking operator contains the leading non-spherical components as well as the usual spherical (angle-averaged) part. We develop a formalism for including the contributions to the effective nucleon-nucleon interaction from the resulting new G-matrix elements that extend the usual two-nucleon spin structure and may not conserve angular momentum. We explore the consequences of parity conservation, time reversal invariance, and nucleon-nucleon antisymmetrization for the new effective interaction. Changes to the calculated cross section and spin observables are small in the energy range from 100 to 200 MeV.

1 Introduction

Proton elastic and inelastic scattering at energies above about 100 MeV are usually described by distorted-wave calculations based on an effective nucleon-nucleon (NN) interaction. When the interaction with the projectile proton is summed, or "folded," over all the nucleons in the target, the resulting potential can be used to model elastic scattering. In addition, the same effective NN interaction becomes the transition potential in the Distorted Wave Impulse Approximation (DWIA) that connects to excited states of the target when the struck nucleon moves to a new shell-model orbit, creating a particle-hole pair. The many-body effects of the nuclear medium are usually incorporated through modifications to this effective NN interaction that depend on the local nuclear density.

Systematic studies [1, 2, 3, 4, 5] have shown that one important contribution to the many-body effects is Pauli blocking, particularly at the lower end of the intermediate energy range. This is the mechanism which prevents nucleons in the nuclear medium from scattering to occupied intermediate states [6]. This restriction is included through a projection operator in the Bethe-Goldstone equation for the G-matrix elements that describe the effective NN interaction inside the nuclear medium.

The usual practice is to average the Pauli projection operator over the intermediate state scattering angle. If, instead of this "spherical" approximation, the non-spherical components are retained, new G-matrix elements appear [7, 8, 9, 10]. While remaining diagonal in total spin S and isospin T, the angular dependence allows coupled G-matrix elements that connect partial wave states where $J \neq J'$ and $\ell \neq \ell'$ beyond the $|\ell - \ell'| = 2$ coupling generated by the tensor interaction. The expanded set of G-matrix elements also depends on M, the magnetic quantum number of the total angular momentum J. In studies that considered the effects on nuclear binding energies, small but non-negligible changes were found when these modifications to the G-matrix were included [8, 9]. It has also been suggested that the spherical approximation is adequate for the central and spin-orbit parts of the effective interaction [7], those pieces that matter the most for elastic scattering and the excitation of natural-parity transitions. But no calculations have been made to check quantitatively in a comparison with scattering measurements how important it is to treat the non-spherical Pauli blocking components.

In a recent calculation of the non-spherical G-matrix [10], we observed that the new couplings in J and ℓ generated only small matrix elements, but the variation of the G-matrix elements with respect to M, the projection of the total angular momentum J, was comparable to the typical size of conventional medium effects. Thus it seemed appropriate to investigate whether these modifications could have an impact on nuclear reactions, and in particular on their spin observables, which are most sensitive to the

non-spherical components of the nuclear force. Proton scattering offers a rich set of polarization observables, including polarization transfer, that reflect the spin dependence of the effective nucleon-nucleon interaction itself.

The dependence on M has been considered previously in calculations of deuteron binding when the deuteron is treated as a projectile traveling through the nuclear medium [11]. While Pauli blocking reduces the deuteron binding energy as the density increases, the amount depends on whether the projection of the deuteron's spin, $|M|$, is 0 or 1 when the quantization axis is taken to be along the direction of motion of the deuteron through nuclear matter. This difference generates a T_P-type momentum-dependent tensor potential in the deuteron-nucleus optical model. Searches for such a potential have not been definitive because of interpretive complications arising from the strong coupling to breakup states [12].

In this paper, we begin with the G-matrix described in Ref. [10]. The important features of the calculation of the G-matrix elements are reviewed in Section II. In order to use this effective interaction in distorted-wave calculations with presently available computer programs, we must transform the G-matrix elements to a coordinate-space representation using a sum of Yukawa functions. In Ref. [5] these coefficients in the Yukawa expansion are fit directly to the values of the G-matrix elements. This is equivalent to earlier methods in which the nucleon-nucleon scattering amplitudes were calculated as a function of scattering angle or momentum transfer and then the Yukawa expansion ranges and coefficients were chosen to best reproduce these angular distributions [13]. For technical reasons, we will follow the second scheme here (see Section III). In the process, we will introduce a multipole expansion of the new G-matrix elements in which the lowest order recovers the result for a spherical Pauli blocking operator. For the spherical Pauli operator, a comparison of the two methods shows practically identical answers.

In Section IV we will compare the non-spherical and spherical treatments of the Pauli blocking operator for representative nuclear transitions at 100 and 200 MeV. We will include a comparison to the free, or density-independent, effective interaction and to an effective interaction that also contains relativistic effects. These comparisons will help gauge the importance of the non-spherical treatment of Pauli blocking relative to density-dependent effects more typically included. We will also include measurements of the cross section and analyzing power to illustrate these effects in relation to the quality of the reproduction of the data. Pauli blocking has its largest effects on the isoscalar central and spin-orbit terms in the effective interaction. These terms are well tested by a comparison to elastic proton scattering or transitions to natural-parity excited states.

Pauli blocking is only one process that is important in the calculation of the effective interaction in the nuclear medium. Others, such as the effects of strong relativistic mean

fields [10, 14, 15] and coupling to Δ-resonances [16], increase the repulsion in the nuclear medium just as does Pauli blocking [17]. In a complete treatment, these should be properly considered, along with the attraction expected to arise from many-body forces. Thus a set of calculations based on the conventional Brueckner-Hartree-Fock approach to nuclear matter only (such as those we present here), should not be expected to provide the final answer. However, a critical evaluation of any of these medium effects requires that the treatment of Pauli blocking not introduce systematic errors large enough to affect our interpretation when agreement with data is considered. We will show that at intermediate energies the inclusion of non-spherical components in the blocking operator produces changes that are modest in size compared to the main density-dependent effects typically included in the effective interaction.

2 Calculation of the G-matrix elements

The Brueckner-Bethe-Goldstone equation [18, 19, 20, 21] describes the scattering of two nucleons in nuclear matter. The presence of the (infinite) nuclear medium is included through Pauli blocking and a mean field arising from the interactions with all the other nucleons. It is convenient to express the momenta of the two nucleons, $\mathbf{k}_1$ for the projectile and $\mathbf{k}_2$ for the struck nucleon, in terms of the relative and center-of-mass motion as

$$\begin{aligned} \mathbf{k} &= (\mathbf{k}_1 - \mathbf{k}_2)/2 \\ \mathbf{P} &= (\mathbf{k}_1 + \mathbf{k}_2)\ . \end{aligned} \tag{1}$$

The total or center-of-mass momentum $\mathbf{P}$ is conserved in the scattering process.

In analogy with free-space scattering, the nuclear matter Bethe-Goldstone equation is given by

$$G(\mathbf{k}', \mathbf{k}, \mathbf{P}, E_0) = V(\mathbf{k}', \mathbf{k}) + \int \frac{d^3\mathbf{k}''}{(2\pi)^3} V(\mathbf{k}', \mathbf{k}'') \frac{Q(\mathbf{k}'', \mathbf{P})}{E_0 + \imath\epsilon - E(\mathbf{P}, \mathbf{k}'')} G(\mathbf{k}'', \mathbf{k}, \mathbf{P}, E_0)\ , \tag{2}$$

where V is the two-body potential. The energy of the two-particle system, E (with E_0 its initial value), includes kinetic energy and the potential energy generated by the mean field. The latter is determined in a separate self-consistent calculation of nuclear matter properties and is conveniently parametrized in terms of effective masses [5]. At this point the direction and magnitude of $\mathbf{P}$ have not been specified.

The Pauli projection operator Q selects intermediate states where both nucleon

momenta lie above the Fermi momentum k_F:

$$Q(\mathbf{k},\mathbf{P},k_F) = \begin{cases} 1 & \text{if } k_1, k_2 > k_F \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Visualizing the (sharp) Fermi surface as a sphere of radius k_F, the condition above imposes the requirement that the tips of the $\mathbf{k}_1$ and $\mathbf{k}_2$ vectors lie outside the sphere. For applications to real nuclei, k_F is treated as a function of the local nuclear density

$$\rho(r) = \frac{2k_F^3(r)}{3\pi^2} \ . \tag{4}$$

The matrix elements for the non-spherical Pauli operator may be written in a partial wave basis as

$$\begin{aligned} &\langle(\ell' S)J'M'|Q(\mathbf{k},\mathbf{P},k_F)|(\ell S)JM\rangle \\ &\quad = \sum_{m_\ell, m_{\ell'}, m_S} \langle \ell' m_{\ell'} S m_S | J'M'\rangle \langle JM|\ell m_\ell S m_S\rangle \langle \ell' m_{\ell'}|Q(\mathbf{k},\mathbf{P},k_F)|\ell m_\ell\rangle \ , \end{aligned} \tag{5}$$

where

$$\langle \ell' m_{\ell'}|Q(\mathbf{k},\mathbf{P},k_F)|\ell m_\ell\rangle = \int d\Omega \ Y^*_{\ell' m_{\ell'}}(\Omega) \ Y_{\ell m_\ell}(\Omega) \ \Theta(|\mathbf{k}_1| - k_F)\Theta(|\mathbf{k}_2| - k_F) \ . \tag{6}$$

The step functions destroy the orthogonality that would otherwise exist for the spherical harmonics in the integral. This allows couplings where $\ell \neq \ell'$ and, through the recoupling coefficients in the summation of Eq. (5), couplings where $J \neq J'$.

If the quantization axis for the projection quantum numbers in Eq. (6) is chosen to lie along $\mathbf{P}$, then there is no dependence on azimuthal direction in the Θ functions, and integration over $d\Omega$ gives $m_\ell = m_{\ell'}$. Because $Q(\mathbf{k},\mathbf{P},k_F)$ is diagonal in m_S, Eq. (5) gives $M = M'$. It then follows from Eq. (2) that G is diagonal in M.

Eq. (5) can be further reduced using standard angular momentum algebra to give

$$\begin{aligned} &\langle(\ell' S)J'M|Q(\mathbf{k},\mathbf{P},k_F)|(\ell S)JM\rangle \\ &\quad = \frac{1}{2}\sum_{L_1}(-1)^{\ell+S+J}\hat{J}'\hat{L}_1^2\hat{\ell}' \ \langle \ell' 0, L_1 0|\ell 0\rangle \ W(J'J\ell'\ell; L_1 S) \\ &\qquad \times \langle J'M, L_1 0|JM\rangle \ \theta_{L_1}(P,k,k_F) \end{aligned} \tag{7}$$

where $\theta_{L_1}(P,k,k_F)$ is the combination of step functions defined in Eqs. (26)-(27) of Ref. [11]. Here and elsewhere we use the notation $\hat{X} = \sqrt{2X+1}$. In Eq. (7) the quantum number M is the projection of $\mathbf{J}$ along the direction of $\mathbf{P}$. We have checked

that values of selected matrix elements calculated with this formula and Eq. (6) agree satisfactorily.

The G-matrix elements are obtained by solving the integral Bethe-Goldstone equation, Eq. (2), in the ℓ, S, J, M, T basis. For any choice of $\mathbf{P}$, the G-matrix elements, $\langle \ell' J' M' | G^{ST}(\mathbf{P}) | \ell J M \rangle$, may be expanded in spherical harmonics $Y_{L\Lambda}(\mathbf{P})$ where L is the angular momentum that recouples J to J'. The coefficients in the recoupling expansion are $G^{LTS}(\ell' J', \ell J, P)$. Thus,

$$\langle \ell' J' M' | G^{ST}(\mathbf{P}) | \ell J M \rangle = \sqrt{4\pi} \sum_{L\Lambda} \langle J' M', L\Lambda | J M \rangle \; \hat{L} \; Y_{L\Lambda}(\mathbf{P}) \; G^{LTS}(\ell' J', \ell J, P) \; , \qquad (8)$$

where the arguments of spherical harmonics are always unit vectors in the direction of the vector indicated. In the limit of a spherically averaged Pauli blocking operator only the $L = 0$ terms in Eq. (8) survive. Equation (8) can be inverted to yield the expansion coefficients

$$G^{LTS}(\ell' J', \ell J, P) = \frac{1}{\hat{J}^2} \sum_{M} \langle J' M, L0 | J M \rangle \; \langle \ell' J' M {:} \mathbf{P} | G^{ST} | \ell J M {:} \mathbf{P} \rangle \; . \qquad (9)$$

where the relation $M = M'$ has been applied. The G-matrix elements on the right hand side are defined with respect to a basis $|\ell J M {:} \mathbf{P}\rangle$ and M refers to the direction of $\mathbf{P}$. For the nuclear matter calculations described here the magnitude of $\mathbf{P}$ has been chosen to be k_1.

Note that if the matrix elements $\langle \ell' J' M : \mathbf{P} | G^{ST} | \ell J M : \mathbf{P} \rangle$ are independent of M and diagonal in J (spherically averaged Pauli blocking) the right hand side of Eq. (9) automatically vanishes unless $L = 0$ because of a property of the C-G coefficients:

$$\sum_{M} \langle J M, L0 | J M \rangle = \hat{J}^2 \delta_{L0}.$$

The anti-symmetrized G-matrix elements are obtained by subtracting from Eq. (9) the same terms but with an additional phase of $(-)^{\ell+S+T}$. This projects out the set of matrix elements with $\ell + S + T$ even, leaving the set that normally describes NN scattering. Due to parity conservation, L takes on only even values. The first step in our transformation to coordinate space was to obtain these L-dependent expansion coefficients. This expansion is expected to converge quickly in L, and the $L = 0$ part was checked for consistency with the result for the spherically averaged Pauli operator.

The G-matrix elements that we will use here were generated from a free-space NN interaction that is a modified version of the Bonn-B potential [17]. Details can be found in Ref. [5]. The model parameters were adjusted so as to achieve a good reproduction of the phase shift analysis results from the Nijmegen group [22] up to 325 MeV.

The number of matrix elements $\langle \ell' J' M{:}\mathbf{P}|G^{ST}|\ell J M{:}\mathbf{P}\rangle$ that are coupled in Eq. (2) increases with M as it becomes possible to incorporate larger values of J. However, the M-dependence decreases with increasing J [10] and we were able to ignore these effects on partial waves with $J, J' > 6$, which were calculated using the usual angle-average approximation.

In general, the new elements of the G-matrix will introduce spin operators that depend on the direction of $\mathbf{P}$, the sum of the projectile momentum $\mathbf{k}_1$ and the momentum $\mathbf{k}_2$ of the particle encountered in the nuclear medium. For practical reasons it is desirable that $\mathbf{P}$ be fixed for a particular transition and incident energy. At the intermediate energies of interest here, when the incident laboratory momentum can be considered large compared with the momenta of the target nucleons, a natural approximation for $\mathbf{P}$ is $\mathbf{k}_1$. We shall show however that this is not a good choice here because it leads to a G-matrix which cannot be expressed in terms of the standard set of spin operators (for example, $\sigma_1 \cdot \sigma_2$, $(\sigma_1 + \sigma_2) \cdot \hat{n}$, $S_{12}(\hat{q})$, and $S_{12}(\hat{Q})$, where the momentum transfer $\hat{q}$ and the normal to the scattering plane $\hat{n}$ establish a coordinate system with $\hat{Q} = \hat{q} \times \hat{n}$). In this paper we choose $\mathbf{P}$ to be parallel to $\mathbf{Q}$ and have magnitude k_1. This does lead to a G-matrix with the standard spin structure. Our choice for the magnitude is a standard one in calculations of medium effects.

3 Transformation of the G-matrix to coordinate space

A calculation of proton elastic or inelastic scattering observables using presently available computer programs requires that the information on the NN interaction in the medium carried by the G-matrix elements be converted into the amplitudes of the effective NN interaction. Usually, the amplitudes for each spin and isospin operator are expanded in a Yukawa series as a function of the momentum transfer. We will begin this section with a description of the general formulas that connect the expanded G-matrix of Eqs. (8) and (9) to NN scattering amplitudes, regardless of the complexity of the coupling. This makes them available for any possible future application. For the calculations we show here, we will restrict ourselves to the set of operators associated with free NN scattering since these are the conventionally used set. After the general formulas, we will show the forms actually used for our calculations.

We first convert from the ℓ, S, J, M, T basis to a representation characterised by definite incoming and outgoing momenta and intrinsic spin projections. The amplitudes in the two bases are related by

$$
\begin{aligned}
\langle \mathbf{k}', S\sigma' | G^T(\mathbf{P}) | \mathbf{k}, S\sigma \rangle &= (2\pi)^3 \sum_{\zeta} \imath^{\ell-\ell'} \langle \ell' m_{\ell'}, S\sigma' | J'M' \rangle \langle \ell m_\ell, S\sigma | JM \rangle \\
&\times \langle J'M', L\Lambda | JM \rangle \; Y_{\ell' m_{\ell'}}(\mathbf{k}') \; Y^*_{\ell m_\ell}(\mathbf{k}) \\
&\times \hat{L}\sqrt{4\pi} \; Y_{L\Lambda}(\mathbf{P}) \; G^{LTS}(\ell' J', \ell J, P) \; ,
\end{aligned}
\tag{10}
$$

with a summation that runs over $\zeta = \ell' J' M' \ell J M m_{\ell'} m_\ell L \Lambda$. The momenta $\mathbf{k}$ and $\mathbf{k}'$ are nucleon momenta in the NN center-of-mass system [see Eq. (1)].

With the normalization factors as in Eq. (10), the partial wave matrix elements $G^{LST}(\ell' J, \ell J, P)$ satisfy unitarity relations which, in the case of uncoupled partial waves in free space, have the form

$$
\Im\left(\frac{1}{G}\right) = \frac{\mu k}{8\pi^2\hbar^2} \tag{11}
$$

where μ is the NN reduced mass. This relation is equivalent to

$$
G_\ell = -\frac{8\pi^2\hbar^2}{\mu k} \; \exp(i\delta_\ell \sin\delta_\ell) \tag{12}
$$

where δ_ℓ is a real NN phase shift. This is the same definition of the partial wave G-matrix elements that was used in Appendix A of Ref. [5].

We denote antisymmetrized matrix elements by $\tilde{G}^T$ where

$$
\langle \mathbf{k}', S\sigma' | \tilde{G}^T(\mathbf{P}) | \mathbf{k}, S\sigma \rangle = 2 \; \langle \mathbf{k}', S\sigma' | G^T(\mathbf{P}) | \mathbf{k}, S\sigma \rangle \; . \tag{13}
$$

For this to be correct, the summation of Eq. (10) and subsequent summations in this paper must restrict ℓ and ℓ' to satisfy

$$
(-1)^{\ell+S+T} = (-1)^{\ell'+S+T} = -1 \; . \tag{14}
$$

For the purpose of applications in DWBA calculations we wish to separate the parts of $\tilde{G}$ associated with the central, spin-orbit, and tensor operators usually used to describe the spin structure of the on-shell ($k = k'$) NN scattering amplitude. In particular, we want to consider the form

$$
\langle \mathbf{k}', S\sigma' | \tilde{G}^T(\mathbf{P}) | \mathbf{k}, S\sigma \rangle = \sum_L \langle \mathbf{k}', S\sigma' | \tilde{G}^{LT}(\mathbf{P}) | \mathbf{k}, S\sigma \rangle ,
$$

where

$$
\begin{aligned}
&\langle \mathbf{k}', S\sigma' | \tilde{G}^{LT}(\mathbf{P}) | \mathbf{k}, S\sigma \rangle = \tilde{G}^{LTS}_C(\theta) \\
&\quad + \delta_{S1} [\tilde{G}^{L1T}_{LS}(\theta)(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\hat{n} + \tilde{G}^{L1T}_{TD}(\theta) S_{12}(\hat{q}) + \tilde{G}^{L1T}_{TX}(\theta) S_{12}(\hat{Q})] \; ,
\end{aligned}
\tag{15}
$$

and where $\vec{\mathbf{q}} = \vec{\mathbf{k}}'_1 - \vec{\mathbf{k}}_1 = \vec{\mathbf{k}}' - \vec{\mathbf{k}}$ is the momentum transfer, $\hat{n}$ is the normal to the scattering plane, and $\hat{Q} = \hat{q} \times \hat{n}$. Each of the $\tilde{G}_i^{LTS}$ is a function of the scattering angle θ. The subscript indicates the spin operator in the NN amplitude corresponding to that coefficient, using C for central (both $S = 0$ and $S = 1$), LS for spin-orbit, and TD and TX for the "direct" and "exchange" parts of the tensor interaction.

Quite generally, the G-matrix $\langle \mathbf{k}', S\sigma' | \tilde{G}^{LT}(\mathbf{P}) | \mathbf{k}, S\sigma \rangle$ can be expanded in terms of the complete set of spin tensors $\tau_{k_S q_S}(S)$ whose matrix elements are

$$\langle S\sigma' | \tau_{k_S q_S} | S\sigma \rangle = \hat{k}_S \ \langle S\sigma, k_S q_S | S\sigma' \rangle, \tag{16}$$

where k_S runs from 0 to $2S$. The coefficients in this expansion are $\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k}, \mathbf{k}')$ and are given by

$$\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k}, \mathbf{k}', \mathbf{P}) = \text{Trace}\ \left(\langle \mathbf{k}', S\sigma' | \tilde{G}^{LT}(\mathbf{P}) | \mathbf{k}, S\sigma \rangle \ \tau_{k_S q_S} \right) . \tag{17}$$

These coefficients $\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k}, \mathbf{k}', \mathbf{P})$ are defined and normalized so that

$$\langle \mathbf{k}', S\sigma' | \tilde{G}^{LT}(\mathbf{P}) | \mathbf{k}, S\sigma \rangle = \hat{S}^{-2} \sum_{k_S q_S} \langle S\sigma' | \tau^{\dagger}_{k_S q_S} | S\sigma \rangle \ \tilde{G}^{LTS}_{k_S q_S}(\mathbf{k}, \mathbf{k}', \mathbf{P}). \tag{18}$$

We emphasize that for a general $\mathbf{P}$ the amplitude $\langle \mathbf{k}', S\sigma' | \tilde{G}^{LT}(\mathbf{P}) | \mathbf{k}, S\sigma \rangle$ of Eq. (13) does not have the form of Eq. (15) but will contain other terms, *e.g.*, $S_{12}(\mathbf{P})$. However, it is shown in Appendix A that if $\mathbf{P}$ is chosen so that it becomes $\mathbf{P}' = \pm\mathbf{P}$ under a rotation of π about $(\mathbf{k}+\mathbf{k}')$, then the form Eq. (15) is sufficiently general. The following formulas will assume that this is the case. The coefficients in Eq. (15) are then given in terms of the $\tilde{G}^{LTS}_{k_S q_S}$ by

$$\tilde{G}^{LTS}_{C}(\theta) = \hat{S}^{-2} \ \tilde{G}^{LTS}_{00}(\mathbf{k}, \mathbf{k}', \mathbf{P}) \tag{19}$$

$$\tilde{G}^{L1T}_{LS}(\theta) = \frac{\sqrt{2\pi}}{6} \sum_{q_S} Y^{*}_{1q_S}(\mathbf{n}) \ \tilde{G}^{L1T}_{1q_S}(\mathbf{k}, \mathbf{k}', \mathbf{P}) \tag{20}$$

$$\tilde{G}^{L1T}_{TD}(\theta) - \frac{1}{2}\tilde{G}^{L1T}_{TX}(\theta) = \frac{\sqrt{10\pi}}{30} \sum_{q_S} Y^{*}_{2q_S}(\mathbf{q}) \ \tilde{G}^{L1T}_{2q_S}(\mathbf{k}, \mathbf{k}', \mathbf{P}) \tag{21}$$

$$-\frac{1}{2}\tilde{G}^{L1T}_{TD}(\theta) + \tilde{G}^{L1T}_{TX}(\theta) = \frac{\sqrt{10\pi}}{30} \sum_{q_S} Y^{*}_{2q_S}(\mathbf{Q}) \ \tilde{G}^{L1T}_{2q_S}(\mathbf{k}, \mathbf{k}', \mathbf{P}) \ . \tag{22}$$

These results follow from the orthogonality of the $\tau_{k_S q_S}$ with respect to the trace operation and the formulas ($S = 1$)

$$\text{Trace}(\tau_{1q_S} \mathbf{S}\cdot\mathbf{A}) = \sqrt{8\pi} \ Y_{1q_S}(\mathbf{A}) \tag{23}$$

$$\text{Trace}(\tau_{2q_S}(3(\mathbf{S}\cdot\mathbf{A})^2 - 2)) = 3\sqrt{\frac{8\pi}{5}} \ Y_{2q_S}(\mathbf{A}) \ , \tag{24}$$

where $\vec{A}$ is an arbitrary unit vector.

Using the partial wave expansion given in Eq. (10) the trace in Eq. (17) can be calculated explicitly to give,

$$\begin{aligned}\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k},\mathbf{k}',\mathbf{P}) &= \sum_{\zeta} \imath^{\ell-\ell'} \hat{S}\hat{J}^2\hat{J}'\hat{k}_\ell\hat{L}(-1)^{J-J'-\ell+k_\ell} \begin{Bmatrix} \ell & \ell' & k_\ell \\ J & J' & L \\ S & S & k_S \end{Bmatrix} \\ &\times \langle k_\ell q_\ell, L\Lambda|k_S q_S\rangle\ \langle \ell' m_{\ell'}, \ell m_\ell|k_\ell q_\ell\rangle \\ &\times Y_{\ell' m_{\ell'}}(\mathbf{k}')\ Y_{\ell m_\ell}(\mathbf{k})\ \sqrt{4\pi}\ Y_{L\Lambda}(\mathbf{P})\ \tilde{G}^{LTS}(\ell' J', \ell J, P)\end{aligned} \tag{25}$$

where the sum runs over $\zeta = \ell' J' \ell J m_{\ell'} m_\ell k_\ell q_\ell \Lambda$ with the restrictions for antisymmetrization noted above.

For our present application, we choose to express the components of these tensors in a right-handed coordinate system with $\hat{z}$ along $\mathbf{k}$ and $\hat{y}$ along $\hat{n}$. We call this the 'standard' coordinate system. We choose $\mathbf{P}$ to be parallel to $(\mathbf{k}+\mathbf{k}')$ so that the restricted form of Eq. (15) is valid. In the limit that the reaction Q-value is small compared to the bombarding energy, the direction of $\mathbf{P}$ is toward $\theta/2$. We also neglect terms with $J' \neq J$ because these matrix elements turn out to be small. Eq. (25) reduces to

$$\begin{aligned}\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k},\mathbf{k}',\mathbf{P}||\mathbf{k}+\mathbf{k}') &= \sum_{\zeta} \imath^{\ell-\ell'} \hat{S}\hat{J}^3\hat{k}_\ell\hat{L}(-1)^{-\ell+k_\ell} \begin{Bmatrix} \ell & \ell' & k_\ell \\ J & J & L \\ S & S & k_S \end{Bmatrix} \\ &\times \langle k_\ell q_\ell, L\Lambda|k_S q_S\rangle\ \langle \ell' q_\ell, \ell 0|k_\ell q_\ell\rangle \\ &\times Y_{\ell' q_\ell}(\theta,0)\hat{\ell}\ Y_{L\Lambda}(\theta/2,0)\ \tilde{G}^{LTS}(\ell' J, \ell J, P)\ ,\end{aligned} \tag{26}$$

The values of $\tilde{G}^{LTS}(\ell' J, \ell J, P)$ were calculated using Eq. (9) in the form

$$\begin{aligned}\tilde{G}^{LTS}(\ell J, \ell J, P) = \frac{1}{\hat{J}^2} \Big\{ &\langle J0, L0|J0\rangle\ \langle \ell' JM=0|\tilde{G}^{ST}|\ell JM=0\rangle \\ &+2\sum_{M>0} \langle JM, L0|JM\rangle\ \langle \ell' JM|\tilde{G}^{ST}|\ell JM\rangle \Big\}\ .\end{aligned} \tag{27}$$

Eq. (26) was split into two summations for the sake of faster computation as

$$
\begin{aligned}
\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k},\mathbf{k}',\mathbf{P}||\mathbf{k}+\mathbf{k}') &= \hat{L}\sum_{J\ell\ell'} i^{\ell-\ell'}\hat{J}^3(-1)^{\ell}\hat{\ell}\hat{S}\ \tilde{G}^{LTS}(\ell' J,\ell J,P) \\
&\times \sum_{k_\ell q_\ell \Lambda} \hat{k}_\ell(-1)^{k_\ell} \begin{Bmatrix} \ell & \ell' & k_\ell \\ J & J & L \\ S & S & k_S \end{Bmatrix} \\
&\times \langle k_\ell q_\ell, L\Lambda | k_S q_S\rangle\ \langle \ell' q_\ell, \ell 0 | k_\ell q_\ell\rangle \\
&\times Y_{\ell' q_\ell}(\theta,0)\ Y_{L\Lambda}(\theta/2,0)\ .
\end{aligned}
\tag{28}
$$

When expressed in terms of the components of the tensors $\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k},\mathbf{k}',\mathbf{P}||\mathbf{k}+\mathbf{k}')$ in the 'standard' coordinate system, Eqs. (19)-(22) reduce to

$$\tilde{G}^{LTS}_{C}(\theta) = \hat{S}^{-2}\ \tilde{G}^{LTS}_{00} \tag{29}$$

$$\tilde{G}^{L1T}_{LS}(\theta) = \frac{i\sqrt{3}}{6}\ \tilde{G}^{L1T}_{11} \tag{30}$$

$$\tilde{G}^{L1T}_{TD}(\theta) = \frac{\sqrt{2}}{72}\left[3(1-\cos\theta)\tilde{G}^{L1T}_{20} + 2\sqrt{6}\sin\theta\ \tilde{G}^{L1T}_{21} + \sqrt{6}(3+\cos\theta)\tilde{G}^{L1T}_{22}\right] \tag{31}$$

$$\tilde{G}^{L1T}_{TX}(\theta) = \frac{\sqrt{2}}{72}\left[3(1+\cos\theta)\tilde{G}^{L1T}_{20} - 2\sqrt{6}\sin\theta\ \tilde{G}^{L1T}_{21} + \sqrt{6}(3-\cos\theta)\tilde{G}^{L1T}_{22}\right]\ , \tag{32}$$

where the θ arguments of the tensor components on the right-hand side have been dropped for simplicity.

An additional transform was needed at the end to replace states where $S = 0, 1$ and $T = 0, 1$ with the singlet and triplet spin and isospin operators customarily used by the distorted-wave programs.

In Ref. [10], it was noted that, if the M-dependent G-matrix elements were simply averaged over M, the result was very close to the one obtained with the standard angle-average calculation. The amplitudes obtained here by retaining only the $L = 0$ term were also observed to reproduce the spherically averaged results to excellent precision. This was used as a check of the computational algorithms.

Values of the density-dependent G-matrix calculated with the spherical Pauli operator were used for partial waves where $7 \leq J \leq 15$, matrix elements that we needed to specify the long-range pion tail of the NN interaction. No partial waves with $J > 15$ were included. The amplitudes of Eqs. (29)–(32) were then calculated at a number of values of θ and reproduced using a sum of Yukawa functions [5,14]. Because the $L = 2$ contributions were much smaller than for $L = 0$, only $L = 2$ was considered and terms with $L \geq 4$ were ignored.

The matrix inversion scheme described in Appendix A of Ref. [5] presumes that the

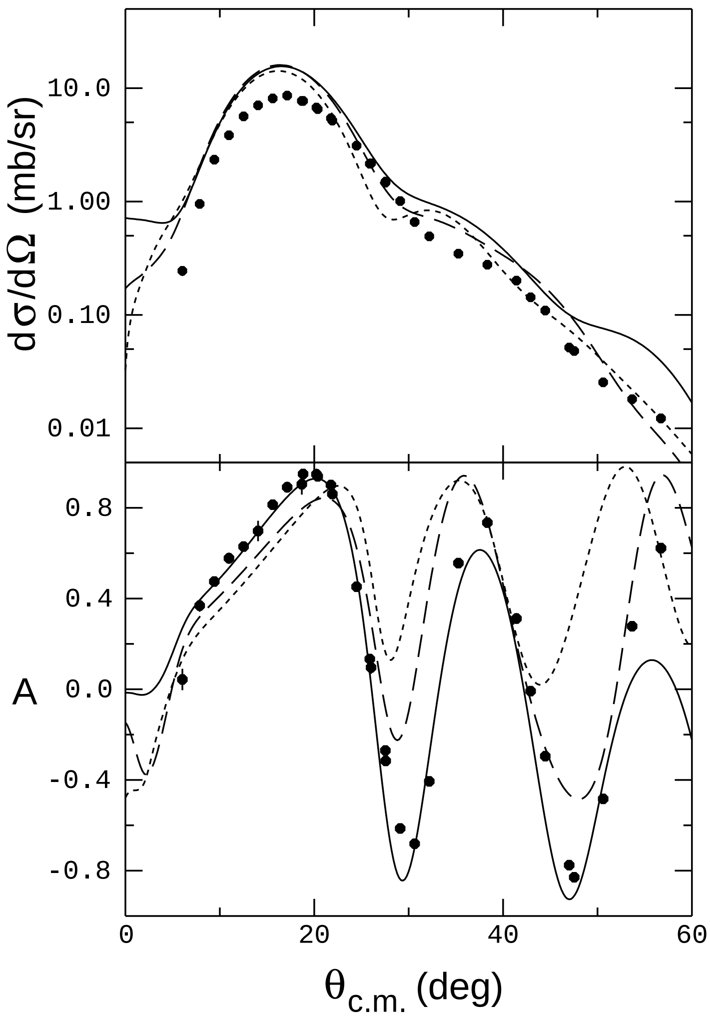
amplitudes associated with the $S_{12}(\hat{q})$ and $S_{12}(\hat{Q})$ are related according to $\tilde{G}^{LST}_{TD}(\pi-\theta) = -(-)^{S+T}\ \tilde{G}^{LST}_{TX}(\theta)$ (as reported for E' and F' in Love and Franey [13]). For our present situation in which $\mathbf{P}$ is chosen to be parallel to $(\mathbf{k}+\mathbf{k}')$, this relationship is no longer valid (see the discussion in Appendix B). So we adopted an older fitting scheme in which the $\tilde{G}^{LST}_{i}(\theta)$ are calculated as a function of scattering angle and reproduced using standard least squares minimization techniques to determine the Yukawa expansion coefficients. Separate coefficients were obtained for $\tilde{G}^{LST}_{TD}(\theta)$ and $\tilde{G}^{LST}_{TX}(\theta)$, leading to the same quality of reproduction usually obtained for the spherical Pauli blocking case.

4 Results for (p,p′) reactions

In the framework referred to by the name of Brueckner-Hartree-Fock (BHF), density-dependent effects on the interaction are included through Eq. (2), with the energy denominator properly modified by the presence of the nuclear matter mean field. Considerations of nucleons moving through nuclear matter as Dirac particles result in what is referred to as the Dirac-Brueckner-Hartree-Fock (DBHF) approach [23]. The latter is known to provide a realistic description of the nuclear matter equation of state [17].

It is useful to gauge the difference between the spherical and non-spherical Pauli operators against the background of these other contributions to medium modifications. To illustrate their effects, in Fig. 1 we show the cross section and analyzing power data for the 3^- state at 3.736 MeV in ^{40}Ca [24]. The beam energy for these measurements was 200 MeV. The short dashed curves contain no density dependence. Including only BHF effects produces the long-dashed curves; DBHF calculations produce the solid curves. As discussed in Ref. [5], the distortions are based on a folded optical potential that was constructed using the same effective interaction as the DWIA transition. The local nuclear matter density was unfolded from the longitudinal electron scattering form factor [25] using the proton charge distribution. The structure formfactors replicate the inelastic electron scattering measurements for this same transition [24]. The calculations were made with the zero-range program LEA [26]. With the treatment of exchange discussed in Appendix B of Ref. [5], the zero-range approximation provides an adequate description of natural parity transitions, as shown by the comparisons illustrated in Fig. 3 of Ref. [5]. Using this method, we can maintain a high quality treatment of the transition formfactor.

The most striking effects of the increasing repulsion in the nuclear medium lie beyond $20°$ where they drive the analyzing power to more negative values and increase the cross section. With DBHF, very good agreement to the analyzing power data is obtained between $8°$ and $35°$ where the cross section is the largest and the DWIA should be at



Stephenson, FIG. 1

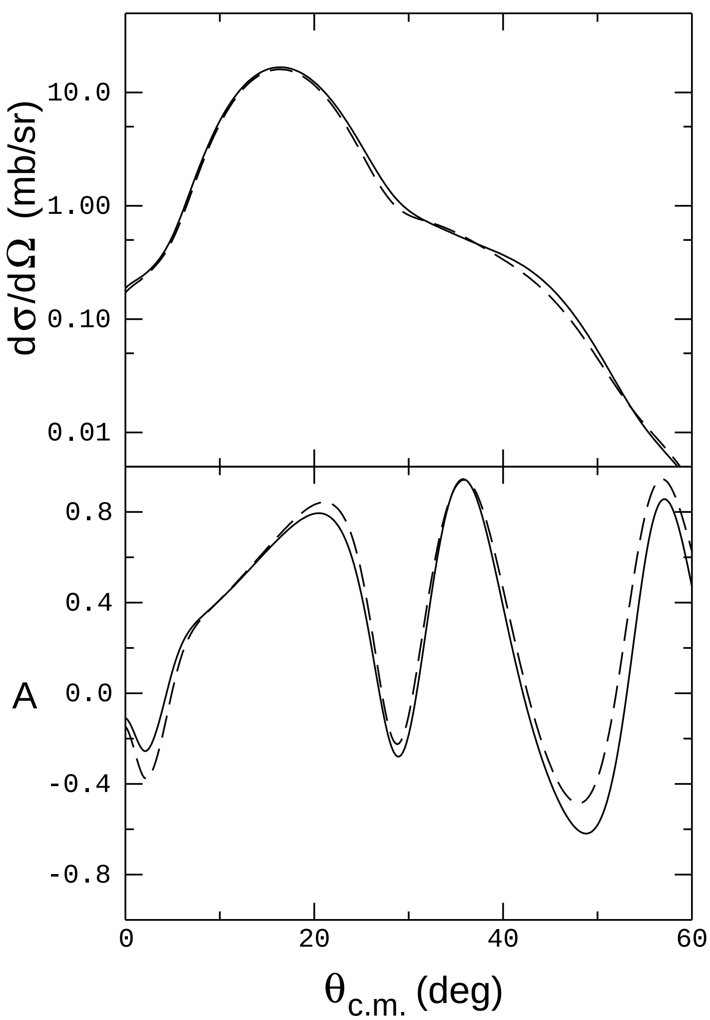
Figure 1: Measurements at 200 MeV of the cross section and analyzing power for the ^{40}Ca(p,p′) transition to the 3^- state at 3.736 MeV. The curves are DWIA calculations that include no medium effects (short dash), spherical BHF effects (long dash), and DBHF effects (solid).

its best. However, the cross section is overestimated by a factor of about 2, a problem of unknown origin with the DWIA that was circumvented for an *empirical* effective interaction by simply decreasing the normalization of the NN amplitudes at all densities [24]. If the calculated cross sections are lowered in this manner to agree with the data at the cross section peak, the DBHF curve provides the best reproduction of the shape of the angular distribution.

In Fig. 2 we show for the same transition BHF-based predictions with non-spherical (solid) and spherical (dashed) Pauli operators. (At this time we do not have DBHF calculations with non-spherical Pauli blocking.) The differences are modest at all angles, being no more than a small fraction of the changes shown in Fig. 1. So given the quality of agreement with the DBHF calculations from Fig. 1, it is not possible to conclude whether the addition of non-spherical components to the treatment of Pauli blocking is required by the data.

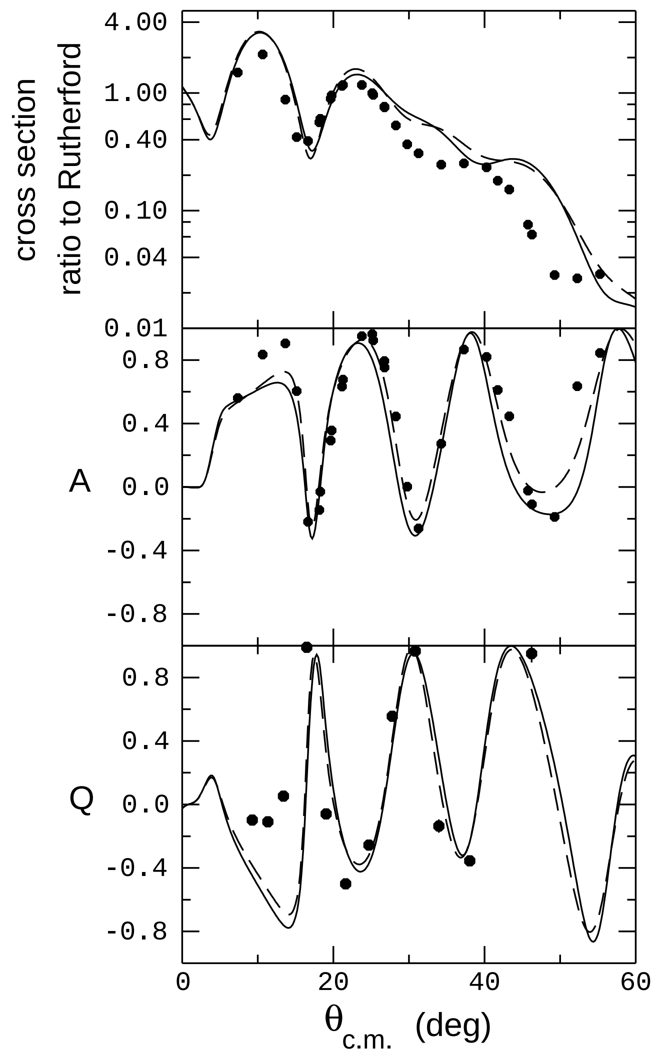
In Fig. 3, the non-spherical (solid) and spherical (dashed) BHF calculations are compared with measurements of the cross section [24], analyzing power [24], and spin rotation parameter $Q(\theta)$ [27] for proton elastic scattering from ^{40}Ca at 200 MeV. To reduce complication, calculations without density dependence and DBHF calculations are not shown. Our earlier results [5] demonstrate that best agreement with elastic scattering is often obtained with just the BHF density dependence, a conclusion that has historically been verified many times [4]. Finite nucleus effects substantially reduce the size of the relativistic medium effects for elastic scattering [28]. This reduction does not take place when the density dependence is obtained through calculations in infinite nuclear matter. Thus agreement between these BHF calculations and elastic scattering is good for both $A(\theta)$ and $Q(\theta)$ in the middle of the angular range of the data. Relativistic effects at angles below 15°, when included, do increase the analyzing power and shift the spin rotation parameter toward more positive values. Thus these differences where the elastic cross section is large can be reduced in the fuller treatment of the medium. The differences between the Pauli operators with and without non-spherical components are again too small to have an impact on the quality of the agreement with experimental data.

Last, it is known that Pauli blocking effects become weaker as the bombarding energy goes up, finally disappearing near the pion production threshold. Figure 4 shows the non-spherical (solid) and spherical (dashed) BHF calculations for the 3^- state in ^{40}Ca at 100 MeV. As expected, the differences are larger than in Fig. 2, but still remain smaller than the changes associated with the baseline medium effects or a DBHF treatment. (Earlier calculations [29] that showed much larger effects at 100 MeV were in error.)



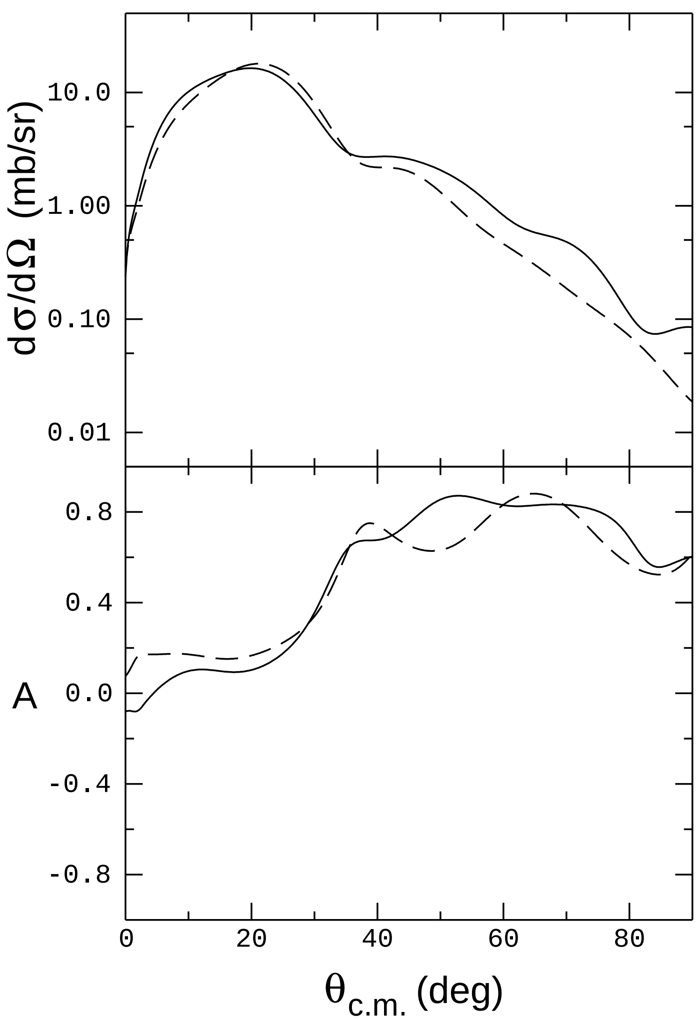
Stephenson, FIG. 2

Figure 2: Comparison of DWIA calculations with spherical (dashed) and non-spherical (solid) Pauli density dependence. The calculations at 200 MeV are for the ^{40}Ca(p,p′) transition to the 3^- state at 3.736 MeV.



Stephenson, FIG. 3

Figure 3: Measurements of the cross section, analyzing power, and spin rotation parameter Q at 200 MeV for proton elastic scattering from ^{40}Ca. The folded optical model calculations are based on density-dependent interactions that include either spherical (dashed) or non-spherical (solid) Pauli effects.



Stephenson, FIG. 4

Figure 4: Comparison of DWIA calculations with spherical (dashed) and non-spherical (solid) Pauli density dependence. The calculations at 100 MeV are for the ^{40}Ca(p,p′) transition to the 3^- state at 3.736 MeV.

5 Conclusions

A number of authors have examined the non-spherical Pauli blocking operator and calculated the changes to the G-matrix elements compared to the spherical approximation. We report here for the first time a formalism that allows the effects of the non-spherical operator to be included in a calculation of nucleon-induced elastic and inelastic scattering. We truncate the resulting G-matrix to remove the elements with $J \neq J'$ or $|\ell - \ell'| > 2$. This truncation involves terms that are small relative to the changes in the G-matrix elements with M, the projection quantum number for J. An expansion is introduced in terms of $\mathbf{L} = \mathbf{J} - \mathbf{J}'$ where the $L = 0$ part essentially recovers the spherically-averaged Pauli blocking result. In this expansion we restrict our consideration to terms with $L < 4$. We wished to look in particular at polarization observables that might be sensitive to the M-dependence of the interaction matrix elements.

Brueckner-Hartree-Fock calculations of the effective interaction including the non-spherical Pauli operator were made at 100 and 200 MeV. They were compared against other BHF-predictions obtained with the more common angle-average approximation, as well as DBHF calculations and those that contained no density dependence at all. The change from spherical to non-spherical in the treatment of the Pauli blocking operator produced changes that were small in comparison to the effects associated with typical BHF or DBHF medium modifications. Given the experimental errors and the likely larger uncertainties in the DWIA theory, the small size of the changes associated with adopting the non-spherical Pauli treatment cannot be deemed significant. These conclusions remained valid with changing bombarding energy. No polarization observables (many were examined) were found to be particularly sensitive to the non-spherical treatment of the Pauli operator. Thus at the level at which present theory can provide an accurate model of nucleon-induced reactions, we find the spherically-averaged treatment of the Pauli projection operator adequate for intermediate-energy proton-nucleus scattering.

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A Symmetries of the G-matrix in infinite uniform nuclear matter

We define the G-matrix as the solution of Eq. (2) and think of it as the set of matrix elements of an operator in the NN relative coordinate space for a fixed $\mathbf{P}$ and k_F. This operator does not conserve the total angular momentum in the NN center-of-mass if $\mathbf{P} \neq 0$. However, the Pauli blocking operator Q is invariant under spatial reflections and time reversal and we will assume that the NN interaction V is also. These symmetries place important restrictions on the free-space G-matrix. Here we examine their consequences for the structure of the on-shell G-matrix in nuclear matter.

If the two-body interaction V is invariant under time reversal, it is straight forward to show formally from Eq. (2) that the $G(\mathbf{P})$ operator for fixed $\mathbf{P}$ satisfies

$$KG(\mathbf{P})K^{-1} = G(\mathbf{P})^{\dagger} \ , \tag{33}$$

where K is the anti-unitary time reversal operator for the system. What this relationship means is that for any states $|\phi\rangle$ and $|\psi\rangle$

$$\langle \phi \mid G(\mathbf{P}) \mid \psi \rangle = \langle K\psi \mid G(\mathbf{P}) \mid K\phi \rangle, \tag{34}$$

where $|K\psi\rangle \equiv K \mid \psi\rangle$.

Our angular momentum states are chosen to have phases such that

$$K \mid (lS)JM\rangle = (-1)^{J+M+\ell} \mid (lS)J-M\rangle \ . \tag{35}$$

The occurence of the phase $(-1)^{\ell}$ in Eq. (35) implies that we have not included $\imath^{\ell}$ factors with the orbital angular momentum wavefunctions $Y_{\ell m}$. This is consistent with what is assumed in Eq. (10).

Using the results of Eqs. (34) and (35) we obtain

$$\begin{aligned} &\langle (l'S)J'M' \mid G^T(\mathbf{P}) \mid (lS)JM\rangle \\ &\quad = \ (-1)^{\ell'+J'+M'+J+\ell+M}\langle (lS)J-M \mid G^T(\mathbf{P}) \mid (l'S)J'-M'\rangle \ . \end{aligned} \tag{36}$$

In the case of no Pauli blocking, G conserves J and M and the G-matrix is independent of M. Together with parity conservation, this leads to the usual result that the G-matrix is symmetric in the angular momentum basis.

When Eq. (36) is used in the definition of G^{LTS} from Eq. (9), we find

$$\tilde{G}^{LTS}(l'J', lJ, P) = (-1)^{J'+\ell'-J-\ell}\frac{\hat{J}'}{\hat{J}}\tilde{G}^{LTS}(lJ, l'J', P) \ , \tag{37}$$

and using this in Eq. (25) (assuming $k = k'$) gives

$$\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k}, \mathbf{k}', \mathbf{P}) = (-1)^{k_S} \tilde{G}^{LTS}_{k_S q_S}(-\mathbf{k}', -\mathbf{k}, \mathbf{P}) \ . \tag{38}$$

Eq. (38) is valid for arbitrary fixed $\mathbf{P}$.

For fixed $\mathbf{P}$ a consequence of parity conservation in Eq. (25) is

$$\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k}, \mathbf{k}', \mathbf{P}) = \tilde{G}^{LTS}_{k_S q_S}(-\mathbf{k}, -\mathbf{k}', \mathbf{P}) \ , \tag{39}$$

Taken together, Eqs. (38) and (39) imply

$$\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k}, \mathbf{k}', \mathbf{P}) = (-1)^{k_S} \tilde{G}^{LTS}_{k_S q_S}(\mathbf{k}', \mathbf{k}, \mathbf{P}) \ . \tag{40}$$

Eqs. (38) and (39) are the generalization of the usual prescriptions for constructing G. They lead to the form given in Eq. (15) in the spherical Pauli-blocking approximation. For a general $\mathbf{P}$, Eqs. (38) and (39) do not restrict G to have the form of Eq. (15) and there will be many other terms involving $\mathbf{P}$, for example $S_{12}(\mathbf{P})$.

We can look at the consequences of Eqs. (38) and (39) in a different way. Eq. (40) is a relationship between two different sets of G-matrix elements corresponding to different ingoing and outgoing momenta with the same (arbitrary) $\mathbf{P}$. We can change this into a relation between matrix elements with the same ingoing and outgoing momenta but with different $\mathbf{P}$'s. We do this by noting that, if $k = k'$ interchanging $\mathbf{k}$ and $\mathbf{k}'$ (as in Eq. (40)) is the same as rotating them by π about $\mathbf{k}+\mathbf{k}'$. Eq. (40) is therefore equivalent to

$$\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k}, \mathbf{k}', \mathbf{P}) = (-1)^{k_S} \tilde{G}^{LTS}_{k_S q_S}(R\mathbf{k}, R\mathbf{k}', R\mathbf{P}') \ , \tag{41}$$

where R denotes the rotation by π about $\mathbf{k} + \mathbf{k}'$ and

$$\mathbf{P}' = R^{-1}\mathbf{P} \ . \tag{42}$$

We next use the fact that the $\tilde{G}^{TS}_{k_S q_S}$ are components of an irreducible tensor of rank k_S, q_S. Therefore the condition Eq. (41), can be written [30]

$$\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k}, \mathbf{k}', \mathbf{P})) = (-1)^{k_S} \sum_{q'_S} \tilde{G}^{LTS}_{k_S q'_S}(\mathbf{k}, \mathbf{k}', \mathbf{P}') \mathcal{D}^{k_S}_{q'_S, q_S}(\mathbf{k} + \mathbf{k}', \pi) \ , \tag{43}$$

where $\mathcal{D}^{k_S}_{q'_S, q_S}(\mathbf{k} + \mathbf{k}', \pi)$ is the rotation matrix corresponding to a rotation of π about $\mathbf{k} + \mathbf{k}'$.

A similar argument based on the parity condition, Eq. (39), and the fact that $-\mathbf{k} = R(\pi, \mathbf{n})\mathbf{k}$ and $-\mathbf{k}' = R(\pi, \mathbf{n})\mathbf{k}'$ gives [30],

$$\tilde{G}^{LTS}_{k_S q_S}(\mathbf{k}, \mathbf{k}', \mathbf{P})) = (-1)^{k_S} \sum_{q'_S} \tilde{G}^{LTS}_{k_S q'_S}(\mathbf{k}, \mathbf{k}', \mathbf{P}'') \mathcal{D}^{k_S}_{q'_S, q_S}(\mathbf{n}, \pi) \ , \tag{44}$$

where $\mathbf{P}'' = R^{-1}(\pi, \mathbf{n})\mathbf{P}$.

In free space, or in the spherical Pauli blocking approximation, the G-matrix does not depend on the direction of $\mathbf{P}$ and Eqs. (43) and (44) become linear relationships between tensor components $\tilde{G}^{LTS}_{k_S q_S}$ of the same rank. Expressed in terms of components in the "standard" coordinate system, Eq. (44) (parity) gives

$$\tilde{G}^{LTS}_{k_S q_S} = (-1)^{k_S + q_S} \tilde{G}^{LTS}_{k_S - q_S}. \tag{45}$$

This simply states that for $k_S = 1$ the vector $\tilde{G}^{LTS}_{1 q_S}$ is proportional to $\mathbf{n}$ and that the $k_S = 2$ tensor is determined by 3 complex amplitudes instead of the general 5:

$$\begin{gathered} \tilde{G}^{LST}_{10} = 0, \quad \tilde{G}^{LST}_{11} = \tilde{G}^{LST}_{1-1} \\ \tilde{G}^{LST}_{20}, \quad \tilde{G}^{LST}_{21} = -\tilde{G}^{LST}_{2-1}, \quad \tilde{G}^{LST}_{22} = \tilde{G}^{LST}_{2-2} \ . \end{gathered} \tag{46}$$

Hence for $S = 1$ a consequence of Eq. (45) in free space is that the G-matrix is determined by 5 complex θ-dependent amplitudes.

Equation (43) (or Eq. (38), time reversal) gives no extra information for $k_S = 1$, but for $k_S = 2$ gives the extra condition

$$\sin\theta(\sqrt{3/2}\,\tilde{G}^{LST}_{20} - \tilde{G}^{LST}_{22}) + 2\cos\theta\,\tilde{G}^{LST}_{21} = 0 \ , \tag{47}$$

so that in fact there are only four independent G-matrix amplitudes. (That Eq. (47) is the only extra condition can easily be seen by expressing Eq. (44) in a coordinate system with z along $\mathbf{k} + \mathbf{k}'$. It is found that Eq. (44) implies $\tilde{G}^{LST}_{21} = 0$ in this system [30].) This is the content of the form given in Eq. (15).

For a general $\mathbf{P}$ in nuclear matter none of these restrictions will be true because Eqs. (43) and (44) relate tensor components with different $\mathbf{P}$. However, the G-operator satisfies $\tilde{G}(-\mathbf{P}) = \tilde{G}(\mathbf{P})$ and hence if

$$\mathbf{P}'' = \pm\mathbf{P} \tag{48}$$

and

$$\mathbf{P}' = \pm\mathbf{P} \ , \tag{49}$$

where $\mathbf{P}'$ and $\mathbf{P}''$ are defined in Eq. (42) and following Eq. (44), respectively, then Eq. (44) does relate tensor components with the same $\mathbf{P}$ and Eqs. (46) and (47), will be satisfied.

This situation occurs when $\mathbf{P}$ lies in the scattering plane; both Eqs. (48) and (49) are satisfied if $\mathbf{P}$ is parallel to $\mathbf{k}+\mathbf{k}'$. This is the choice we make in this paper. With this choice the conditions, Eqs. (46) and (47), are satisfied and, equivalently, the G-matrix has the form of Eq. (15) on shell.

B Direct and exchange tensor amplitudes for arbitrary P

We can ensure that the amplitudes $\tilde{G}_i^{LST}(\theta)$ of Eq. (15) are anti-symmetrized by subtracting in each case the same amplitude with the additional phase factor $(-)^{S+T}$ and with $\mathbf{k}$ replaced by $-\mathbf{k}$. This requires that any value of ℓ that appears in a partial wave decomposition of $\tilde{G}_i^{LST}(\theta)$ satisfies $(-)^{\ell+S+T} = -1$.

The direction of $\mathbf{P}$, the total momentum of the colliding nucleons defined in Eq. (1), remains the same for both terms in the anti-symmetrization subtraction. If we consider just the amplitudes associated with the tensor spin operators $S_{12}(\hat{q})$ and $S_{12}(\hat{Q})$, then in terms of the unsymmetrized amplitudes $G_i^{LST}(\theta)$:

$$\langle \mathbf{k}'|G(\mathbf{P})|\mathbf{k}\rangle = G_{TD}^{LST}(\cos\theta, \mathbf{P}\mathbf{q}, \mathbf{P}\mathbf{Q}, \mathbf{P}\mathbf{n})\, S_{12}(\hat{q}) + G_{TX}^{LST}(\cos\theta, \mathbf{P}\mathbf{q}, \mathbf{P}\mathbf{Q}, \mathbf{P}\mathbf{n})\, S_{12}(\hat{Q}) \quad (50)$$

where the dot products indicate any scalar generated with the vector $\mathbf{P}$. The amplitude to be subtracted is

$$\begin{aligned}\langle \mathbf{k}'|G(\mathbf{P})|-\mathbf{k}\rangle &= G_{TD}^{LST}(-\cos\theta, \mathbf{P}\cdot\mathbf{Q}, \mathbf{P}\cdot\mathbf{q}, -\mathbf{P}\cdot\mathbf{n})\, S_{12}(\hat{Q}) \\ &+ G_{TX}^{LST}(-\cos\theta, \mathbf{P}\cdot\mathbf{Q}, \mathbf{P}\cdot\mathbf{q}, -\mathbf{P}\cdot\mathbf{n})\, S_{12}(\hat{q})\ .\end{aligned} \quad (51)$$

Note that the definition of the scattering angle becomes the complement of the original value. With the reversal of the direction of $\mathbf{k}$, the vectors $\mathbf{q}$ and $\mathbf{Q}$ have interchanged places and magnitudes while $\mathbf{P}$ remains fixed. Thus, the direction of $\hat{n} = \hat{q}\times\hat{Q}$ reverses.

By combining terms, the anti-symmetrized form $G_i^{LST}(\theta)$ is given by

$$\tilde{G}_{TD}^{LST}(\theta) = G_{TD}^{LST}(\cos\theta, \mathbf{P}\cdot\mathbf{q}, \mathbf{P}\cdot\mathbf{Q}, \mathbf{P}\cdot\mathbf{n}) - (-)^{S+T}\, G_{TX}^{LST}(-\cos\theta, \mathbf{P}\cdot\mathbf{Q}, \mathbf{P}\cdot\mathbf{q}, -\mathbf{P}\cdot\mathbf{n}) \quad (52)$$

for the coefficient of $S_{12}(\hat{q})$ and

$$\begin{aligned}\tilde{G}_{TX}^{LST}(\theta) \;=\; & G_{TX}^{LST}(\cos\theta, \mathbf{P}\cdot\mathbf{q}, \mathbf{P}\cdot\mathbf{Q}, \mathbf{P}\cdot\mathbf{n}) \\ & -(-)^{S+T}\, G_{TD}^{LST}(-\cos\theta, \mathbf{P}\cdot\mathbf{Q}, \mathbf{P}\cdot\mathbf{q}, -\mathbf{P}\cdot\mathbf{n}) \\ & -(-)^{S+T}\, \Big[G_{TD}^{LST}(-\cos\theta, \mathbf{P}\cdot\mathbf{Q}, \mathbf{P}\cdot\mathbf{q}, -\mathbf{P}\cdot\mathbf{n}) \\ & -(-)^{S+T}\, G_{TX}^{LST}(\cos\theta, \mathbf{P}\cdot\mathbf{q}, \mathbf{P}\cdot\mathbf{Q}, \mathbf{P}\cdot\mathbf{n})\Big]\end{aligned} \quad (53)$$

for the coefficient of $S_{12}(\hat{Q})$.

For NN scattering, the anti-symmetrized amplitudes satisfy [13]

$$\tilde{G}_{TD}^{LST}(\pi-\theta) = -(-)^{S+T}\, \tilde{G}_{TX}^{LST}(\theta)\ . \quad (54)$$

This permits the same Yukawa expansion coefficients to be used for $\tilde{G}^{LST}_{TD}(\theta)$ and $\tilde{G}^{LST}_{TX}(\theta)$ provided that Q is used in place of q in the Yukawa expansion of the TX term [13]. For the non-spherical calculations used in this paper, $\mathbf{P}$ is along the direction of $\hat{Q}$, thus the dot products involving $\mathbf{Q}$ in Eqs. (52) and (53) remain. Due to the presence of different arguments in the TD and TX amplitudes, Eq. (54) is not necessarily satisfied. This is the case for the calculations reported here.

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